Exercise Set #27
Answers to Odd-Numbered Exercises

27-1 (a) Since the hypothesis test concerns the comparison of the mean for more than two populations with independent random samples, a one-way ANOVA should be performed.

(b) Step 1
H₀: \( \mu_A = \mu_B = \mu_C \)
H₁: At least one of the means is different.
\( \alpha = 0.01 \)

Step 2
\( n_A = 6, \; \bar{x}_A = 110.0, \quad n_B = 4, \; \bar{x}_B = 107.5, \quad n_C = 5, \; \bar{x}_C = 106.0 \)

\( f_{2, 12} = 1.86 \)

Step 3
do not reject \( H_0 \)
0.10 < \( p \)-value

Step 4
Since \( f_{2, 12} = 1.86 \) and \( f_{2, 12; 0.01} = 6.93 \), we do not have sufficient evidence to reject \( H_0 \). We conclude that there is no difference among these three methods in the mean amount of impurities left after purification (0.10 < \( p \)-value). Since \( H_0 \) is not rejected, no further analysis is needed.

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>( f )</th>
<th>( p )-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods</td>
<td>45</td>
<td>2</td>
<td>22.500</td>
<td>1.86</td>
<td>0.10 &lt; ( p )-value</td>
</tr>
<tr>
<td>Error</td>
<td>145</td>
<td>12</td>
<td>12.083</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>190</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(c) multiple box plots - Since amount of impurities left after purification (ppm) is a quantitative variable, three box plots, one for each method, is an appropriate graphical display.
(d) Since $H_0$ is not rejected, the Type II error is possible, which is concluding that $\mu_A = \mu_B = \mu_C$ when actually at least one mean is different.

(e) $H_0$ would not have been rejected with $\alpha = 0.05$ and with $\alpha = 0.10$.

(f) The presence of one or more outliers in the data would suggest that the $f$ statistic may not be appropriate.

(g) Data where SSB is equal to zero, but SSE is not equal to zero.

<table>
<thead>
<tr>
<th>Impurity Levels (ppm) for Three Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Method A: 106 107 108 108 109 110</td>
</tr>
<tr>
<td>Method B: 107 108 108 109</td>
</tr>
<tr>
<td>Method C: 106 107 108 109 110</td>
</tr>
</tbody>
</table>

In order for SSB to be equal to zero, the sample means must all be equal to each other. In order for SSE not to be equal to zero, there must be some difference between observations within the same sample. The data displayed is one possible answer.

Data where SSE is equal to zero, but SSB is not equal to zero.

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In order for SSE to be equal to zero, observations within the same sample must all be equal to each other. In order for SSB not to be equal to zero, there must be at least one difference among sample means. The data displayed is one possible answer.
(b) **Step 1**

- $H_0: \mu_{A-B} = 0$
- $H_1: \mu_{A-B} > 0$
- $\alpha = 0.05$ (one-sided)

**Step 2**

- $n = 18$, $d = 32.9444$, $s_d = 60.1053$, $t_{17} = +2.325$

**Step 3**

- Reject $H_0$
- $0.01 < p\text{-value} < 0.025$

Since $t_{17} = 2.325$ and $t_{17; 0.05} = 1.740$, we have sufficient evidence to reject $H_0$. We conclude that there is an increase in mean milk sales after the series of commercials promoting milk consumption ($0.01 < p\text{-value} < 0.025$).

(c) **Rejecting $H_0$** suggests that there is an increase in mean milk sales after the promotion but gives us no information about the size of the increase. A confidence interval will provide information about the size of the increase. Using $t_{17; 0.025} = 2.110$, the limits of the 95% confidence interval are $3.055 < \mu_{A-B} < 62.834$.

We are 95% confident that the mean increase in milk sales is between 3.055 and 62.834 units.
(d) five-number summary: \(-35, 0, +11, +40, +200\)

Since there are potential outliers, and the box plot looks skewed, there may be some concern about whether the \(t\) statistic is appropriate.

(e) Since \(H_0\) is rejected, the Type I error is possible, which is concluding that \(\mu_{A-B} > 0\) when actually \(\mu_{A-B} = 0\).

(f) \(H_0\) would not have been rejected with \(\alpha = 0.01\) but would have been rejected with \(\alpha = 0.10\).