1. Let \( X_1, X_2, ..., X_n \) be a random sample of size \( n \) from a Poisson distribution with mean \( \theta \).

(a) Prove that \( X_1 + X_2 + ... + X_n \) has a Poisson distribution, and find the mean.

(b) Prove that \( 2X_1 \) does not have a Poisson distribution.

2. Suppose \( X_1, X_2, ..., X_n \) is a random sample from a Normal(\( \mu \), \( \sigma^2 \)) distribution. Let \( Q = (n - 1)S^2 / \sigma^2 \).

(a) What kind of distribution does \( Q \) have, and why?

(b) Find \( E(S^6) \), and simplify it to a form which involves no gamma functions.

3. Let \( X_1, X_2, ..., X_8 \) be a random sample from a distribution with p.d.f.

\[ f(x) = (x + 1) / 2 \quad \text{if} \quad -1 < x < 1. \]

(a) Find the exact value of \( P(X_i > 0) \) for each \( i = 1, 2, ..., 8 \).

(b) Use the Central Limit Theorem to approximate \( P(\bar{X} > 0) \).
4. Let $X_1, X_2, ..., X_n$ be a random sample from a Normal(10, 4) distribution.

(a) Suppose $n = 2$. Find $P(3X_1 - 4X_2 < 0)$.

(b) Suppose $n = 100$. First, find $P(X_i > 10.88)$ for each $i = 1, 2, ..., 100$; then use the Central Limit Theorem to approximate $P(Y < 35)$, where

$$Y = \text{the number of } X_i\text{s larger than 10.88}$$

5. Suppose $Y$ is a random variable such that $E(Y) = 15$ and $\text{Var}(Y) = 0.25$.

(a) Find an upper bound for $P(|Y - 15| \geq 2)$.

(b) Find a lower bound for the probability that $17 < Y + 3 < 19$.

6. Suppose $X$ has a $b(n, p)$ distribution.

(a) If $p = 0.74$, what is the smallest $n$ for which we can be reasonably certain that this binomial distribution can be approximated by a normal distribution?

(b) If $p = 0.074$ and $n = 50$, what distribution should be used to approximate this binomial distribution?
7. Suppose that \( X_1, X_2, \ldots, X_{30} \) is a random sample from a Normal(0,1) distribution. Name the type of distribution each of the following random variables has:

(a) \[ \sum_{i=1}^{5} X_i^2 \]

(b) \[ 3 \sum_{i=1}^{5} X_i^2 \quad \text{and} \quad 5 \sum_{i=6}^{8} X_i^2 \]

(c) \[ 4X_1 \quad \text{and} \quad \sqrt{\sum_{i=2}^{17} X_i^2} \]
8. Find each of the following.

(a) a constant c such that \( P(T < c) = 0.10 \) where \( T \) has a Student's \( t \) distribution with 8 degrees of freedom.

(b) a constant c such that \( P(|T| < c) = 0.99 \) where \( T \) has a Student's \( t \) distribution with 13 degrees of freedom.

(c) a constant c such that \( P(F > c) = 0.95 \) where \( F \) has a Fisher's \( f \) distribution with 8 numerator degrees of freedom and 3 denominator degrees of freedom.

(d) a constant c such that \( P(F > c) = 0.05 \) where \( F \) has a Fisher's \( f \) distribution with 7 numerator degrees of freedom and 9 denominator degrees of freedom.

9. Suppose that \( X_1, X_2, \ldots, X_n \) is a random sample from a chi-square distribution with two degrees of freedom. Let \( Y = X_1 + X_2 + \ldots + X_n \). Use the Central Limit Theorem to find constants \( k_1 \) and \( k_2 \) such that the limiting distribution of

\[
\frac{Y - k_1}{k_2}
\]

as \( n \) goes to \( \infty \) is the standard normal.
Here are some additional practice exercises from the textbook (with answers in the back of the textbook):

6.1-1, 6.1-3, 6.1-5, 6.1-7, 6.1-9, 6.1-11,

6.2-3a, 6.2-5, 6.2-9, 6.2-13, 6.2-15, 6.2-19,

6.3-1, 6.3-3, 6.3-5, 6.3-7(a), 6.3-9, 6.3-11,

6.4-1, 6.4-3, 6.4-7, 6.4-9,

6.5-1ab, 6.5-3, 6.5-5, 6.5-7, 6.5-9, 6.5-17,

6.7-1,

6.8-1, 6.8-5, 6.8-7